Universidade do Minho
Escola de Ciências
Centro de Matemática

Centro de Matemática da Universidade do Minho

Seminário de Análise

Date :: 2013 February 13,2:20pm

Place :: Seminars Room of DMA (B4009), Campus of Gualtar

Speaker :: Assis Azevedo, CMAT, Universidade do Minho

Title :: Dynamics of a quasi-quadratic map

Abstract :: We consider the map $\chi : \mathbb{Q} \to \mathbb{Q}$ given by $\chi(x) = x\lceil x \rceil$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x, and study the problem of finding, for each rational, the smallest number of iterations of χ that eventually sends it into an integer. Given two natural numbers M and n, we prove that the set of irreducible fractions with denominator M whose orbits by χ reach an integer in exactly n iterations is a disjoint union of congruence classes modulo M^n, establishing along the way a finite procedure to ascertain them. We also describe an efficient algorithm to decide if an orbit fails to hit an integer until a prescribed number of iterations, and deduce that the probability that an orbit enters $\mathbb{Z}$ is equal to one.

Joint work with Maria Carvalho and António Machiavelo, Centro de Matemática da Universidade do Porto.