Universidade do Minho
Escola de Ciências
Centro de Matemática

Date	::	2012 November 02, 14h
Place	::	Seminars Room of DMA (B4009), Campus of Gualtar
Speaker	::	L. Lévi, Pau University, Laboratory of Applied Mathematics
Title	::	On a Mathematical Analysis for a Class of Hyperbolic - Parabolic Problems
Abstract	::	Based on the works in [1, 2, 3], this talk deals with the well-posedness for the coupling of multidimensional quasilinear diffusion-transport equations. Namely, let Ω be a bounded domain Ω of $\mathbb{R}^n$, $n \geq 1$, such that $\overline{\Omega} = \overline{\Omega_h} \cup \overline{\Omega_p}$; Ω_h and Ω_p being two disjoint bounded domains with Lipschitz boundaries respectively Γ_h and Γ_p. We are interested in the existence and uniqueness for the boundaries values problem:

For any $T > 0$, find a measurable and bounded function u on $Q = (0, T) \times \Omega$ such as

$$\begin{cases} \partial_t u + \mathrm{d}iv_x(f_p(u)\nabla P(x)) + g_p(t, x, u) & = & \Delta\phi(u) & \text{in } Q_p = (0, T) \times \Omega_p, \\ \partial_t u + \mathrm{d}iv_x(f_h(u)\nabla P(x)) + g_h(t, x, u) & = & 0 & \text{in } Q_h = (0, T) \times \Omega_h, \\ u & = & 0 & \text{on } \Sigma \equiv (0, T) \times \partial\Omega, \\ u(0, .) & = & u_0 & \text{on } \Omega, \end{cases}$$

subject to the transmission conditions along the interface $\Gamma_{hp} = \Gamma_h \cap \Gamma_p$:

$$(f_p(u)\nabla P - f_h(u)\nabla P)) \cdot \nu_h = \nabla\phi(u) \cdot \nu_h \text{ on } \Sigma_{hp} \equiv (0, T) \times \Gamma_{hp}.$$

We first provide the definition of a *weak entropy solution*. For the uniqueness two situations are investigated and whether the characteristics of the first order operator set on Ω_h are entering or outgoing along the interface. The existence result is obtained through a vanishing viscosity method on the hyperbolic zone.

[1] G.Aguilar, L.Lévi and M.Madaune-Tort, Coupling of Multidimensional Parabolic and Hyperbolic Equations. *J. Hyperbolic. Differ. Equ.* **3**, 1 (2006) 53-80.

[2] G.Aguilar, L.Lévi and M.Madaune-Tort, Nonlinear Multidimensional Parabolic-Hyperbolic Equations. 2006 International Conference in honor of Jacqueline Fleckinger. *Electron. J. Diff. Eqns.* Conference 16 (2007), pp. 15-28.

[3] J.Jimenez and L.Lévi, A Mathematical Analysis for some Class of Hyperbolic-Parabolic Problems. *Advances in Mathematical Sciences and Applications* (2010)