SOME RESULTS ON CONSERVATIVE AND ON HAMILTONIAN DYNAMICS

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Abstract. In this seminar we will briefly discuss some recent results on conservative and on Hamiltonian dynamics.

Consider a C^1-divergence-free vector field X defined on a closed, connected Riemannian manifold. We say that X is a C^1-star vector field if any divergence-free vector field in some C^1-neighborhood of X has all singularities and all closed orbits hyperbolic. We will discuss the equivalence between the following conditions:

- X is a C^1-star vector field.
- X is in the C^1-interior of the set of *expansive* divergence-free vector fields.
- X is in the C^1-interior of the set of divergence-free vector fields which satisfy the *shadowing property*.
- X has no singularities and X is *uniformly hyperbolic*.

Now, let H be a *Hamiltonian* defined on a symplectic manifold M, $e \in H(M) \subset \mathbb{R}$ and $\mathcal{E}_{H,e}$ a connected component of $H^{-1}(\{e\})$ without singularities. A *Hamiltonian system*, say a triplet $(H, e, \mathcal{E}_{H,e})$, is *uniformly hyperbolic* if $\mathcal{E}_{H,e}$ is uniformly hyperbolic. A Hamiltonian system $(H, e, \mathcal{E}_{H,e})$ is a *Hamiltonian star system* if all the closed orbits of $\mathcal{E}_{H,e}$ are hyperbolic and the same holds for a connected component of $\tilde{H}^{-1}(\{\tilde{e}\})$, close to $\mathcal{E}_{H,e}$, for any $\tilde{H}$ in some C^2-neighborhood of H and for any $\tilde{e}$ in some neighborhood of e. In this context, we show that a *Hamiltonian star system* defined on a 4-dimensional symplectic manifold is *uniformly hyperbolic*.

To finish the seminar, we will discuss the Hamiltonian version of a result proved by Bonatti and Crovisier for diffeomorphisms, whereby a C^1-generic conservative diffeomorphism is transitive.

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