AN INTRODUCTION TO LARGE DEVIATIONS

ADRIANA NEUMANN

ABSTRACT. The theory of large deviations studies rare events, these are events whose probability is very small. Studying events whose probability decays exponentially is important to understand the asymptotic behaviour of the model, because we understand which are the possible deviations from the expected behaviour. To illustrate this, consider a simple example. Let $\{X_n\}_{n\in\mathbb{N}}$ be a sequence of independent and identically distributed random variables, taking values in $\mathbb{R}$, with zero average and $\mathbb{E}[e^{\lambda X_1}] < \infty$, $\forall \lambda \in \mathbb{R}$. We know that for all $[a,b]$ such that $0 \notin [a,b]$, we have $\mathbb{P}[\frac{1}{n}\sum_{k=0}^{n} X_k \in [a,b]] \to 0$, when n increases to infinity. But, we would like to know what is the rate of convergence, in other words, we would like to find a function $I : \mathbb{R} \to \mathbb{R} \cup \{\infty\}$ such that $\mathbb{P}[\frac{1}{n}\sum_{k=0}^{n} X_k \in [a,b]] \sim e^{-n \inf\{I(x),\, x\in[a,b]\}}$. This theory has been applied to a wide range of processes such as random walks, particle systems, where it is necessary to obtain information about the system.

In this mini-course I will focus on simple ideas, nevertheless powerful ideas behind large deviations, stated in a non-technical scenario, and on its applications to the random walk in the discrete torus $\mathbb{Z}/k\mathbb{Z}$, see [3] and [1]. In the proof of the large deviations for the random walk it will appear the main ideas of the proof of the large deviations for the exclusion process, which is a stochastic process that has been extensively studied in the last years, see [2].

REFERENCES

[1] Freidlin, M. I.; Wentzel, A. D.: *Random Perturbations of Dynamical Systems*, Springer, (1991).

[2] Kipnis, C.; Landim, C.: *Scaling limits of interacting particle systems*. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], 320. Springer-Verlag, Berlin, (1999).

[3] Lopes, A.; Neumann, A.: *Large Deviations via Aubry-Mather Theory*, work in progress, (2014).