

Moore-Penrose invertibility in involutory rings: the case

$$aa^\dagger = bb^\dagger$$

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Abstract

In this paper, we consider Moore-Penrose invertibility in rings with a general involution. Given two von Neumann regular elements a, b in a general ring with an arbitrary involution, we aim to give necessary and sufficient conditions to $aa^\dagger = bb^\dagger$. As a special case, EP elements are considered.

References

- [1] Castro González, N.; Koliha, J. J.; Wei, Yimin; Perturbation of the Drazin inverse for matrices with equal eigenprojections at zero. *Linear Algebra Appl.* 312 (2000), no. 1-3, 181–189.
- [2] Hartwig, Robert E.; Block generalized inverses. *Arch. Rational Mech. Anal.* 61 (1976), no. 3, 197–251.
- [3] Hartwig, Robert E.; How to partially order regular elements. *Math. Japon.* 25 (1980), no. 1, 1–13.
- [4] Hartwig, Robert E.; Styan, George P. H.; On some characterizations of the “star” partial ordering for matrices and rank subtractivity. *Linear Algebra Appl.* 82 (1986), 145–161.

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- [5] Koliha, J. J.; Djordjević, Dragan; Cvetković, Dragana; Moore-Penrose inverse in rings with involution; *Linear Algebra Appl.* 426 (2007), no. 2-3, 371–381.
- [6] Koliha, J. J.; Patrício, Pedro; Elements of rings with equal spectral idempotents; *J. Aust. Math. Soc.* 72 (2002), no. 1, 137–152.
- [7] Lam, T.Y.; *A First Course in Noncommutative Rings*, Springer-Verlag, New York, 1991.
- [8] Patrício, Pedro; The Moore-Penrose inverse of von Neumann regular matrices over a ring. *Linear Algebra Appl.* 332/334 (2001), 469–483.
- [9] Patrício, Pedro; The Moore-Penrose inverse of a factorization. *Linear Algebra Appl.* 370 (2003), 227–235.
- [10] Patrício, Pedro; Puystjens, Roland; Generalized invertibility in two semigroups of a ring. *Linear Algebra Appl.* 377 (2004), 125–139.
- [11] Puystjens, Roland; Robinson, Donald W.; The Moore-Penrose inverse of a morphism with factorization. *Linear Algebra Appl.* 40 (1981), 129–141.