MINICOURSE ON STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

ERIC GAUTIER (CREST - ENSAE, PARIS)

This is a 10 hours introductory course for PhD students given in the Universidade do Minho (Braga) between September 24 and October 5 2012.

The aim of the course is to provide an introduction to Stochastic Partial Differential Equations (SPDE for short). We will consider stochastic evolution equations of the form

$$\frac{\partial u}{\partial t} = Au + f(u) + g(u)\dot{W}$$

with corresponding initial and boundary conditions, where u is a function of the time and space variables, A is an unbounded operator (for example the Laplacian in the case of the stochastic heat equation), f is a nonlinearity and $g(u)\dot{W}$ is a notation for some (possibly multiplicative) noise. We will review some useful results on stochastic calculus and stochastic differential equations and present two approaches to stochastic integration in infinite dimension: the random field and the semi group approaches. We will then introduce the various types of solutions for SPDEs and some results regarding existence and uniqueness of mild solutions. Based on a few specific examples we will study the qualitative influence of the noise on the behavior of the solutions including blow-up, ergodicty and large deviations.

References

- DA PRATO, G., AND ZABCZYK, J. (1992) Stochastic Equations in Infinite Dimensions, Encyclopedia of Mathematics and its Applications, Cambridge University Press.
- [2] WALSH, J. B. (1986), An Introduction to Stochastic Partial Differential Equations, in P.L. Hennequin, Editor, Ecole d'été de probabilités de Saint-Flour, XIV 1984, LNM 1180, 236-439, Springer Verlag.
- [3] ZAMBOTTI, L. Stochastic Partial Differential Equations, http://www.proba.jussieu.fr/pageperso/zambotti/spde3.pdf.