The initial value problem for a system of high order nonlinear Schrödinger equations

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Abstract: We investigate some well-posedness issues for the initial value problem (IVP) associated to the system

$$\begin{cases} 2i\partial_t u + q\partial_x^2 u + i\gamma\partial_x^3 u + 2i\beta\left(|u|^2 + \sigma_\beta |w|^2\right)\partial_x u + 2\alpha u(|u|^2 + \sigma_\alpha |w|^2) \\ +2i\mu u\partial_x\left(|u|^2 + \sigma_\mu |w|^2\right) = 0 \\ 2i\partial_t w + q\partial_x^2 w + i\gamma\partial_x^3 w + 2i\beta\left(|w|^2 + \sigma_\beta |u|^2\right)\partial_x w + 2\alpha w(|w|^2 + \sigma_\alpha |u|^2) \\ +2i\mu w\partial_x\left(|w|^2 + \sigma_\mu |u|^2\right) = 0. \end{cases}$$

This system describes the dynamic of two nonlinear short-optical pulses envelope $u(x,t)$ and $w(x,t)$ in fibers. It was derived by Porsezian, Shanmugha Sundaram e Mahalingam in 1994, and generalizes the model derived by Hasegawa-Kodama in 1985. In this work we study local well-posedness results for the IVP with data in Sobolev spaces $H^s(\mathbb{R})\times H^s(\mathbb{R})$, $s \geq 1/4$ and in the periodic case in $H^s(\mathbb{T})\times H^s(\mathbb{T})$, $s \geq 1/2$. We show global well-posedness results for the system with data in Sobolev spaces $H^s(\mathbb{R})\times H^s(\mathbb{R})$, $3/5 < s \leq 1$ and $H^1(\mathbb{T})\times H^1(\mathbb{T})$ in the particular case $\sigma_\alpha = \sigma_\beta = \sigma_\mu = 1$. We also obtained ill-posedness result for the IVP with data in Sobolev spaces $H^s(\mathbb{R})\times H^s(\mathbb{R})$,$-1/2 < s < 1/4$. To prove this last result, we apply the splitting argument introduced by Kenig, Ponce, Vega and Bourgain .